

UNITED STATES DISTRICT COURT  
SOUTHERN DISTRICT OF NEW YORK

FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

JPMORGAN CHASE & CO., et al.,

Defendants.

**Declaration Of Charles D.  
Cowan, Ph.D. In Response To  
Declaration Of Arnold Barnett,  
Ph.D.**

11 Civ. 6188 (DLC)

FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

HSBC NORTH AMERICA HOLDINGS, INC., et  
al.,

Defendants.

11 Civ. 6189 (DLC)

FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

BARCLAYS BANK PLC, et al.,

Defendants.

11 Civ. 6190 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

DEUTSCHE BANK AG, et al.,

Defendants.

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11 Civ. 6192 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

FIRST HORIZON NATIONAL CORP., et al.,

Defendants.

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11 Civ. 6193 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

BANK OF AMERICA CORP., et al.,

Defendants.

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11 Civ. 6195 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

CITIGROUP INC., et al.,

Defendants.

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11 Civ. 6196 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

GOLDMAN, SACHS & CO., et al.,

Defendants.

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11 Civ. 6198 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

CREDIT SUISSE HOLDINGS (USA), INC., et al.,

Defendants.

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11 Civ. 6200 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

NOMURA HOLDING AMERICA, INC., et al.,

Defendants.

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11 Civ. 6201 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

MERRILL LYNCH & CO., INC., et al.,

Defendants.

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11 Civ. 6202 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

SG AMERICAS, INC., et al.,

Defendants.

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11 Civ. 6203 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

MORGAN STANLEY, et al.,

Defendants.

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11 Civ. 6739 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

ALLY FINANCIAL INC., et al.,

Defendants.

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11 Civ. 7010 (DLC)

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FEDERAL HOUSING FINANCE AGENCY, AS  
CONSERVATOR FOR THE FEDERAL  
NATIONAL MORTGAGE ASSOCIATION AND  
THE FEDERAL HOME LOAN MORTGAGE  
CORPORATION,

Plaintiff,

-against-

GENERAL ELECTRIC COMPANY, et al.,

Defendants.

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11 Civ. 7048 (DLC)

## I. Introduction

1. I submit this declaration in support of Plaintiff Federal Housing Finance Agency's ("FHFA") Opposition to Defendants' Preliminary *Daubert* Motion to Exclude the October 10, 2012 Expert Report of Charles D. Cowan, Ph.D. (the "Second Cowan Report") and in response to the October 26, 2012 Declaration of Arnold Barnett in support of that motion (the "Barnett Declaration").

2. Dr. Barnett makes a series of basic errors in his Declaration, which I describe below.

## II. Dr. Barnett Miscalculated The Margins Of Error

3. Dr. Barnett asserts that I have miscalculated the margins of error for all sample values other than 50 percent and has supplied his own calculations. Barnett Decl. ¶ 26. Dr. Barnett is mistaken. To derive his accusations, Dr. Barnett relies on the following formula:

$$e = Z \sqrt{\frac{P(1-P)}{n-1} * \left(1 - \frac{n}{N}\right)}$$

4. This, however, is an approximation of the exact formula, and a particularly bad approximation for smaller samples and whenever the defect rate (P in this formula) is near extremes (i.e., near zero or near 100).<sup>1</sup> To see why, I offer Charts 1 and 2. Chart 1 shows an example of the correct distribution for a sample of size 100 and a low defect rate, in this case 5%.

5. Only integers (0, 1, 2, etc.) are possible in the calculation here, because a loan either has a defect or it does not. The number of defective loans also can only be a non-negative integer, because a loan without a defect is not counted. I thus did not use this approximation, but

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<sup>1</sup> See, e.g., EMANUEL PARZEN, MODERN PROBABILITY THEORY AND ITS APPLICATIONS 245 (John Wiley & Sons, Inc. 1960); and WILLIAM G. COCHRAN, SAMPLING TECHNIQUES 56-59 (3rd ed., John Wiley & Sons, Inc. 1977).

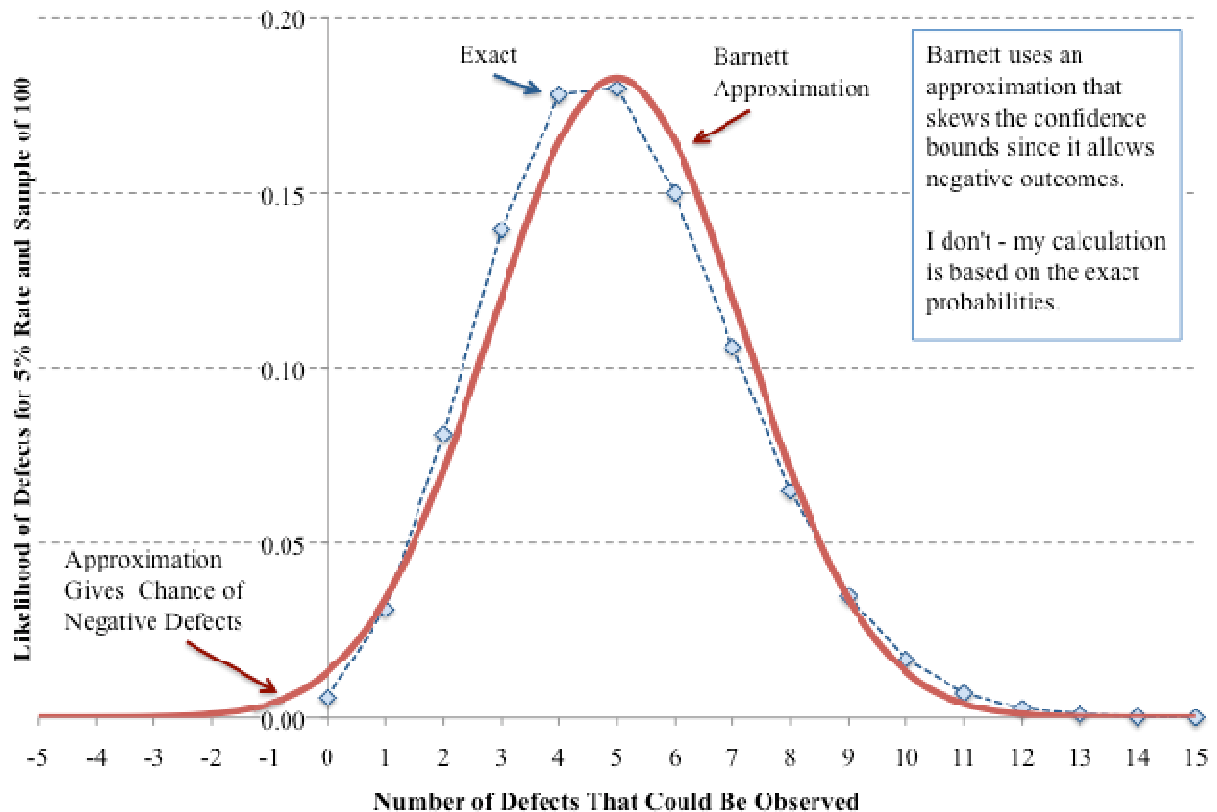
rather the exact formula;  $B(x | n, LB) = \text{Confidence Level } \alpha$ , where  $B$  is the cumulative binomial distribution (as shown in Chart 1),  $x/n$  is the estimated defect rate,  $n$  is 100; and I solve for the value where there is only  $\alpha / 2$  chance that I could obtain this result (namely,  $x/n$ ) from a distribution where the sample size is 100 and the probability of defect is  $LB$ .<sup>2</sup> In other words,  $x$  can only be a non-negative integer. I do not allow the lower confidence bound to be negative (because the number of defective loans cannot be negative) or to apply to non-integers, which are not measurable in the sample (because the number of defective loans must be an integer, as there are no partial loans).

6. The approximation used by Dr. Barnett, however, allows all values between each of the integers (.1, .2, .3, etc.) and *negative* values for the number of defects. In fact, there is a substantial probability associated with negative defect counts in Dr. Barnett's approximation, even though they are impossible.

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<sup>2</sup> Cochran does this explicitly "[w]hen the normal approximation does not apply," giving an example for when a sample is of size 100. Cochran, op. cit. p. 59.

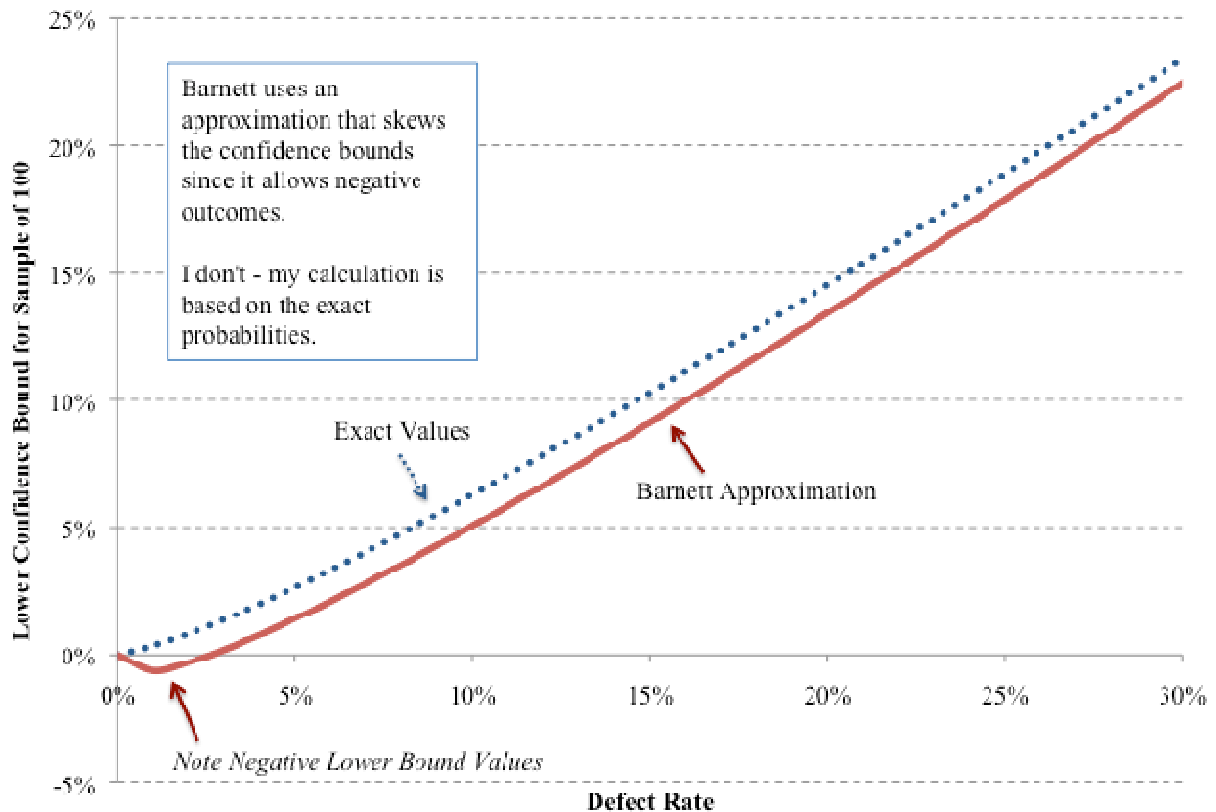
**Chart 1: Probabilities of the Number of Defects Observed in a Sample of Size 100 with a Defect Rate of 5%**



7. While this is an excellent approximation for much larger samples, it does not work well for smaller samples, which is another reason why I did not use this approximation. From Chart 1, the reader can see that the left portion of the distribution, where the lower confidence bound is (the point where there is only a 5% probability in the left portion of the curve), is pulled left from what one would see in the exact distribution. In fact, the 95% lower confidence bound can be negative using this faulty approximation, even though the defect rate can never be negative. This is seen in Chart 2.



**Chart 2: Lower Confidence Bound in a Sample of Size 100 for all Defect Rates between 0% and 30%**



8. For a sample size of 100, the lower 95% confidence bound is negative for any defect rate below 4%. Using Dr. Barnett's approximation, for a sample size of 10, the lower 95% confidence bound is negative for any defect rate below 28% and can achieve a value as low as -9%. Dr. Barnett's approximation thus produces significant errors for small sample sizes and for lower defect rates.

9. One other consideration is the direction of investigation. There are arguments to be made that the proper analysis considers only whether the defect rate is low, not also consideration of much higher values. In this way, the testing is asymmetrical, and the confidence range should be also, with maximum weight given to where the risk is greatest. Excellent

examples of determination of such a confidence interval, where all weight is given to risk above or below a certain bound, are found in Cochran and also in audit sampling textbooks.<sup>3</sup>

10. In addition, for all defect rates in his report, Dr. Barnett's approximation overstates the margin of error for small sample sizes, as here.

### **III. My Sampling Design Does Not Call For Clustering**

11. My sampling design defines the relevant population as the relevant SLGs for each Securitization. That is, for Securitizations for which the GSEs purchased Certificates backed by different SLGs, I treat the population as those SLGs together. This affects the 37 Securitizations for which the GSEs purchased Certificates backed by different SLGs. It does not affect the other 412 Securitizations.

12. This definition of the relevant population was part of the instructions I received. Second Cowan Report ¶ 50. Dr. Barnett asserts that this practice is "cluster[ing]." Barnett Decl. ¶ 12. He is mistaken. Clustering refers generally to a methodology for grouping similar or geographically contiguous objects in such a way that it is possible to select the clusters as units in the sample instead of individual units or loans.<sup>4</sup> I did not do this.

### **IV. The Underwriting Review Will Provide A Binary Result**

13. Dr. Barnett criticizes my sampling protocol because it is designed for a binary (e.g., yes or no) result from the re-underwriting review. This criticism is unsound, because Dr. Barnett is confusing a summary of the reliability for a binary outcome with the design. The design works for any type of extrapolation. Further, the relevant inquiry is indeed binary: it is whether the representations Defendants made about the mortgage loans were true. Second

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<sup>3</sup> Cochran, op. cit. pp. 59-60; *see also* HERBERT ARKIN, HANDBOOK OF SAMPLING FOR AUDITING AND ACCOUNTING 141 (3rd ed., Prentice Hall 1984).

<sup>4</sup> *See* STEVEN K. THOMPSON, SAMPLING 129 (2d ed., John Wiley & Sons, Inc. 2002); Cochran, op. cit. Chs. 9-11.

Cowan Report ¶ 34. The result can only be binary: yes or no. Defendants raise “exceptions” to underwriting guidelines as a possible third category. Dfds’ Mem. at 14. But that is not a third category at all. The only question is whether disputed underwriting goes into the breach or non-breach category.

14. I can also measure the difference the “disputed” breaches make. I can do this by placing the “disputed” breaches in the breach category and projecting the sample to the population, and then I can place the “disputed” breaches in the non-breach category and project the sample to the population. This allows me to measure the effect of the disputed breaches, exactly the analysis Dr. Barnett says is impossible. Barnett Decl. ¶ 11.

#### **V. FICO Is A Reasonable Stratification Variable**

15. Dr. Barnett’s criticism of my choice of stratification variable is empty. First, stratification can only improve (that is, decrease) the margin of error, as Dr. Barnett admits. Thus, even if my choice of stratification variable turns out not to be good, it will not affect the representativeness of my samples or decrease the precision of my estimate. It will merely not deliver a benefit. Furthermore, what Dr. Barnett fails to mention is that the only time that stratification will not decrease the margin of error is when the percent of breaches is exactly the same in all strata. For 100 loans, 25 to a stratum, this means that the number of breaches would have to be identical in all four strata (all 0, all 1, all 2, etc.). This happens 26 ways (zero to 25) out of 456,976 possible outcomes; in 456,950 possible outcomes, the margin of error does improve.

16. Second, Dr. Barnett does not suggest a different stratification variable to use from those available. His criticism therefore is not of my choice, but rather of the available stratification variables.

17. Third, FICO is a reasonable stratification variable. Contrary to Defendants' assertions, FICO is likely to be correlated with underwriting breaches and false statements concerning borrower characteristics. Defendants do not dispute that FICO score is correlated with likelihood of default. Dfids' Mem. at 20. They also must concede that originators have no need to breach their underwriting guidelines for a borrower with a high FICO score, and further that borrowers with high FICO scores have little or no incentive to make false statements on their loan applications, because borrowers with high FICO scores tend to qualify for loans under the originators' underwriting guidelines. It necessarily follows that lower FICO scores will correlate with underwriting guideline breaches and other false statements by both the originator and the borrower, because those are the borrowers who might not otherwise qualify for loans and thus would require underwriting guideline breaches and other false statements in order to be extended such loans. In addition, FICO score presents other advantages over the other potential stratification variables. First, FICO score is correlated with other variables on the loan tapes. Second, FICO score is less likely to be misrepresented than other available variables. It is, therefore, a reasonable stratification variable to use.

**VI. It Makes No Difference What Value Is Assigned Where FICO Score Is Missing, So Long As It Is Consistent**

18. If the loans are missing FICO scores, there is no evidence that the loans should fall into any quartile in the stratification design. As loans with missing FICO scores are still relevant to the sampling process, they have to be placed into one of the strata. Further, a missing FICO score could be a flag that the underwriter (who was required to obtain a credit score report) was trying to hide the status of the borrower. Finally, it makes no sense to try to apply a fictitious FICO score to a borrower. We want to identify and focus on borrowers with no FICO score because they are more likely to have a breach. Putting borrowers into the lowest FICO

group, the one which would be expected to have the highest breach rate, is perfectly consistent with what is being studied.

19. Dr. Barnett's further argument regarding the inclusion of "invalid" FICO scores is unsound. Missing FICO scores, labeled with a zero, were treated as "missing." Loans with zero FICO scores were included in the first stratum and were sampled in this way. Each stratum had 25% of the population in the stratum. Sampling proportionately, each stratum in the sample has 25% of the sample. Each stratum in the sample and each stratum in the population is exactly 25% of the total. Any test of representativeness on FICO score according to the stratum groupings shows that FICO score in the sample is exactly representative of the population.

## **VII. Choice Of Extrapolation Method Can Be Based On Diminution Of The Margin Of Error**

20. Dr. Barnett asserts that "the margin of error is determined by an inflexible formula. One cannot manipulate that formula, or do anything to 'minimize' the margin of error." Barnett Decl. ¶ 9. He therefore asserts that one can never, contrary to my assertion, choose an extrapolation method based on a reduction in the margin of error. *Id.* He is wrong. Post-stratification is a technique often used for reduction in the margin of error, as described, for example, in Dr. Barnett's textbook of choice, Thompson.<sup>5</sup>

21. As an example, suppose that breaches occur for larger loans, and that all loans of more than \$100,000 have a material defect, and all loans of less than \$100,000 have no defects. In this example, if 50% of the loans in the population had original balances greater than \$100,000, then the breach rate estimate would be 50%. But there would be no sampling variance because the estimate for greater than \$100,000 is 100% breach, the estimate for less than \$100,000 is 0% breach, and both of these values have no variability associated with them. Thus,

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<sup>5</sup> Thompson, op. cit. pp. 124-25.

it is easy to imagine a situation where one can readily minimize the margin of error by efficiently using what is observed in the data once the sample is reviewed.

### **VIII. The Potential For Missing Loan Files Does Not Preclude Proper Sample Design**

22. Dr. Barnett states that because I have “not provided details as to how he would handle missing loan files other than by purporting to reserve the right to supplement his samples, Dr. Cowan has not provided the ultimate samples. Without knowing the ultimate samples, it is not possible to determine whether such samples would be representative or randomly selected.” Barnett Decl. ¶ 8(f).

23. This is a non sequitur. Even if there are missing loan files, the sample will still have been randomly selected and will therefore be representative of the population. What Dr. Barnett appears to mean is that, as a result of missing loan files, the loan files available from the sample may not match the population. But this is not surprising. In every research investigation problems arise with the availability and suitability of data. This is not a reason not to sample, but rather can be dealt with through proper advance planning. If Defendants and other third parties cannot produce loan files, supplementation of the samples can occur by sampling in exactly the same way the first batch was sampled to provide enough loans for review. Random supplementation of a random sample is also representative of the population from which both samples are selected. Most real-world (as opposed to theoretical) investigations are fraught with data difficulties.<sup>6</sup>

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<sup>6</sup> Thompson, *op. cit.* Chs. 23-26. *See also* LESLIE KISH, SURVEY SAMPLING Part II “Special Problems and Techniques” (John Wiley & Sons, Inc. 1995).

**IX. The Sampling Protocol Can Be Used To Assess The Relationship Between Breaches And Loan Performance And Can Be Used To Generate Other Quantifications Of Harm**

24. Dr. Barnett asserts that my “samples likely would be insufficient to perform other data analyses, such as the impact of breach on loan performance. The samples may also be inadequate to address other questions the parties may determine to be relevant to the litigation, such as loss causation and/or damages.” Barnett Decl. ¶ 8(g). He is wrong. My samples can be used to assess the impact of breaches on loan performance and thus can be used for damages calculations relating to that issue, if that is what is legally required.

25. For the impact of the breach on loan performance, it is easy to construct an example where a test can show that the breached loans defaulted more frequently, using only the sample.

	<u>Defaulted</u>	<u>Not Defaulted</u>	<u>Total</u>	<u>Rate</u>	<u>Difference</u>	<u>Likelihood Difference is Zero</u>
Breached	11	39	50	22.0%	14.0%	4.99%
<u>Not Breached</u>	<u>4</u>	<u>46</u>	<u>50</u>	<u>8.0%</u>		
Total	15	85	100	15.0%		

26. For a relatively low default rate in the population, we can easily detect a small but still material difference (14%) in default rates (not defaulted would include current and prepays). There is less than a five percent chance that the rates for breached and non-breached loans have the same default rate in this example. Larger differences are even easier to measure. Smaller differences may be possible to measure in post-stratified analyses.

27. All of the FHFA cases have multiple offerings at issue. I can use all of the offerings for analysis of loan performance, as well as the individual offerings. When I collapse the multiple offerings, I have much more powerful analyses and can detect much smaller

differences. The sampling protocol can also calculate a dollar estimate of harm from a sample of loans, if that is called for.

28. Dr. Barnett is also wrong in his assertion “[a]dditionally, for questions that do not receive a binary yes/no classification, there is no ‘maximum margin of error’ that can be derived in advance.” Barnett Decl. ¶ 11. I show, in Appendix A, that there is a simple formulation for this.

**X. All Of The Re-Drawn Samples Come Exclusively From The Relevant Supporting Loan Groups**

29. Dr. Barnett asserts that four of the loans in the re-drawn ACE 2006-OP2 sample are not from group 1, the relevant Supporting Loan Group. Barnett Decl. ¶ 8(f). I have re-examined the re-drawn sample and the loan tape from ACE 2006-OP2. The number of loans in the relevant SLG, group 1, exactly matches the number of loans identified in the prospectus supplement. According to the loan tape provided by Defendants, all loans in the sample were selected from group 1.

**XI. Dr. Barnett’s Inability To Replicate My Results Is Not Due To Any Lack Of Data**

30. Dr. Barnett claims he cannot in all cases replicate my samples. Barnett Decl. ¶ 28. I do not know why he cannot do so. I have provided the samples, the loan data I used to draw them, and the computer programs I used to draw the samples, which include the starting point for the fixed sequence of the random numbers known as the “seed.” With these materials, Dr. Barnett should be able to re-create exactly the samples I have drawn.

31. Dr. Barnett also claims he cannot replicate the results of my representativeness testing. Barnett Decl. ¶ 34. But I have provided him with the formulas I used and the data on which the tests were run. *See* Second Cowan Report. He thus should be able to re-create those tests exactly. In addition, Dr. Barnett’s statement is misleading for two reasons. First, while Dr.



Barnett states that he cannot reproduce my results exactly because of the wide variety and conditions under which Chi-square and Z-tests are performed, he does nearly replicate the results without using my tests, with the values having the same order of magnitude and all  $p$ -values being within one percentage point of each other, a truly remarkable consistency. Second, Dr. Barnett's own tests, provide stronger support—in all but one instance—for the conclusions I have drawn for representativeness. *Compare* Barnett Decl. Exs. B-D with Second Cowan Report Appx. 2. Both Dr. Barnett and I express our results as  $p$ -values. The higher the  $p$ -value is, the stronger the agreement between the sample and the population is, which is the test for representativeness. For each test, except ABFC 2007-WMC1,<sup>7</sup> Dr. Barnett's results show higher  $p$ -values than mine do. *Id.* His tests therefore provide stronger support for my conclusions as to representativeness than mine do.

## **XII. Dr. Barnett's Other Criticisms Are Without Merit**

32. Dr. Barnett claims that using a sample designed for a binary outcome cannot address outcomes for continuous variables. This is also wrong. As an example, suppose that we obtain Combined-Loan-to-Value (CLTV) both from the loan tapes and also through the re-underwriting of the loan files. I can take a difference between these values and determine if it is positive, negative, or zero on average. If the CLTV from re-underwriting is on average greater than the CLTV on the loan tapes, this is a measure of increased risk across all loans in the population. At the same time, I may only be interested in whether the CLTV from re-underwriting is greater than the CLTV from the loan tape. This is a binary outcome. Either can be measured with no difficulty, each adding some value to different parts of the investigation by both Plaintiff and the Defendants.

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<sup>7</sup> Barnett Decl. Ex. 4.

33. Dr. Barnett claims that other variables could have been used for testing the representativeness of the sample and focuses on geography. However, Dr. Barnett does not state how he would test representativeness: by census region (of which there are four), census division (of which there are nine), state (50, 51, or 52 depending on the treatment of the District of Columbia and Puerto Rico), or some other measure. For a sample of 100, the distribution of loans among the 50+ states results in only 2 loans on average per state, which would require collapsing some states, thus introducing additional problems. Dr. Barnett does not suggest a solution to these problems.

34. Finally, Defendants state that, at times, there are multiple Supporting Loan Groups within an offering and that these may have different characteristics described in the Prospectus Supplements. My sampling design can take this into account by using the counts and total dollar values on the loan tapes to compute weighted averages of the statements in the Prospectus Supplements.

Declared under penalty of perjury  
this 9th day of November 2012

A handwritten signature in black ink, reading "Charles D. Cowan". The signature is written in a cursive, flowing style.

CHARLES D. COWAN, Ph.D.

## Appendix A: Proofs Regarding Maximum Variability And Estimation Of Sample Size

From Cochran, op. cit. p. 153, I have the formula for extrapolation:

$$\text{TPB} = \frac{\text{UPB in Defective Loans in Sample}}{\text{UPB in All Loans in Sample}} * \text{UPB in All Loans in Population} = \frac{d}{b} B$$

Where UPB is unpaid principal balances plus interest owed, lower case values are total from the sample, and upper case values that are known from the loan tapes for the population. Summarize Total Principal Balance of Defective Loans as TPB.

The formula for the margin of error for a total like TPB is similar to the one cited by Dr. Barnett, but now looks like:

$$e = Z\sqrt{\text{Variance}(\text{TPB})}$$

We want to show that there is a “worst case” and also that there is a maximum error. From Cochran, op. cit. p. 154, I have the formula for the variability of the extrapolation:

$$\text{Variance}(\text{TPB}) = \frac{N^2}{n} \left( S_d^2 + \left( \frac{D}{B} \right)^2 S_b^2 - 2\rho \left( \frac{D}{B} \right) S_d S_b \right)$$

where  $\rho$  is the correlation between D and B. The value  $\rho$  must be positive because D and B both measure exactly the same values, except that D is sometimes zero for non-defective loans.

From the loan tapes, I know  $S_b$  exactly. I can approximate  $S_d$  by recalling that the numerator is the sum of values b for defective loans plus zero for loans that are not defective. So a close estimate of D from the sample would be  $P*B$ , where P is the proportion of defects. This will be exactly correct if defects are unrelated to loan balance, and conservative if larger loans are more likely to have defects. If B is fixed and P is the variable, then

$$S_d^2 = \text{Variance}(P * B) = B^2 V(p) = B^2 P(1 - P)$$

and the  $\text{Variance}(\text{TPB})$  can be rewritten as

$$\text{Variance}(\text{TPB}) = \frac{N^2}{n} \left( B^2 P(1 - P) + \left( \frac{P * B}{B} \right)^2 S_b^2 - 2\rho \left( \frac{P * B}{B} \right) B P(1 - P) S_b \right)$$

I am now in a situation very similar to the solution to the “worst case” analysis for sampling for an estimate of P. I know N, B, and  $S_b$ , and I can set  $\rho$  to zero to obtain the worst case variance. Now I simply maximize the abbreviated formula:

$$\text{Variance (TPB)} = \frac{N^2}{n} \left( B^2 P(1 - P) + P^2 S_b^2 \right)$$

through differentiation to find that:

$$P = \frac{1}{2} \frac{B^2}{S_b^2 - B^2}$$

This value of P is the “worst case” to design for, with the special case that as  $B^2$  gets very large relative to  $S_b^2$ , P approaches .5, the “worst case” scenario for the liability evaluation. Thus, designing the sample for liability also supports estimation of a continuous variable for this type of ratio estimate.

Finally, in terms of knowing the maximum variability, we can substitute this revised estimate for P, where the worst case occurs, into the abbreviated Variance(TPB) to find the maximum variability under very conservative assumptions.